# Exercise 21

If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$.

---

### Solution

Differentiate both sides of the given equation with respect to $x$.

$$\frac{d}{dx}\{f(x) + x^2[f(x)]^3\} = \frac{d}{dx}(10)$$

$$\frac{d}{dx}[f(x)] + \frac{d}{dx}\{x^2[f(x)]^3\} = 0$$

$$f'(x) + \left[\frac{d}{dx}(x^2)\right][f(x)]^3 + x^2\left\{\frac{d}{dx}[f(x)]^3\right\} = 0$$

$$f'(x) + (2x)[f(x)]^3 + x^2\left\{3[f(x)]^2 \cdot \frac{d}{dx}[f(x)]\right\} = 0$$

$$f'(x) + 2x[f(x)]^3 + 3x^2[f(x)]^2 f'(x) = 0$$

Solve for $f'(x)$.

$$f'(x)\{1 + 3x^2[f(x)]^2\} = -2x[f(x)]^3$$

$$f'(x) = -\frac{2x[f(x)]^3}{1 + 3x^2[f(x)]^2}$$

Evaluate it at $x = 1$.

$$\begin{aligned} f'(1) &= -\frac{2(1)[f(1)]^3}{1 + 3(1)^2[f(1)]^2} \\ &= -\frac{2(1)(2)^3}{1 + 3(1)^2(2)^2} \\ &= -\frac{16}{13} \end{aligned}$$